

TABLE OF FAMILIES OF ALTERNATING KNOTS WITH THEIR CONWAY'S FUNCTION

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Dans le centenaire d'Henri Poincaré 1854-1912

Abstract

A table of the families of alternating knots formed by conways is presented. The Conway's function is shown with the use of linear algebra in terms of natural numbers, called conways, that represent the number of crossings along a direction, as it was used by J. Conway for the classification of knots.

Colored figures and tangles show the parts of the knots or tangles with a definite handedness: all the colored parts of the knot family are associated to a particular orientation. For example all the colored conways have a right hand screw thread, and all the white conways have the opposite handedness. Figures for six conways were colored with two different colors for forty two families in order to show the dissection of the knot in two tangles corresponding to a particular factorization of the Conway's function. The Conway's function of each family is expressed as the internal product of two vectors corresponding to each of two colored family 2-tangles, and with a full factorization which is not unique.

1 INTRODUCTION

This document presents a table of the 65 families of prime alternating knots (see links also) formed by conways, from one to six conways. The conway is represented in these figures by an "eye" with a a_j inside it. Such a conway represent a tangle formed by the twisting of two strands which in the projection to form the figures has a_j crossings. The a_j 's represent a natural number labeled with j . Each figure of whatever family in this table has as a figure caption the Conway's function of the family. This terminology gives credit to the seminal paper presented in 1967 and published in 1970 by J. Conway [1].

The Conway's function of a family of knots is a polynomial with coefficients one, formed by monomials which are products of conways. It correspond to the sum of the absolute values of the coefficients of the Alexander polynomial which Conway highlights for each knot or link in his tabulation of the simplest knots [1]. The number of monomials of the Conway's function is the Conway's number of the seed, which is the knot corresponding to the family that has $a_j = 1$ for all the conways of the family.

This table reproduces the same figures in [2] where all the terminology not fully defined in the present introduction is explained and discussed. The difference with this document and [2] is the emphasis because here we present only the figures of the different prime families of knots. A detailed account is found in [2]. Another difference is that the Conway's functions in the figure caption of the figure of each different family appears here in terms of an explicit matrix product of vectors and matrices associated to 2-tangles or 3-tangles, highlighting the stellar presence of the matrix metric in the 2-tangle case,

$$M \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (1)$$

and the metric matrix in the 3-tangle case

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (2)$$

The full explanation of the table requires reference to [2], however I believe it is very useful to have in the web this document independently

of the multiple definitions and properties of the algebra in [2]. The factorizations presented in this table show in an evident form many (but not all) of the possible representations of the Conway's function of a particular family.

The Conway's function may be presented in all the cases as the interior product of two vectors corresponding to 2-tangles. For 35 families formed by 6 conways we give the Conway function as an interior product of two 2-tangles formed each by 3 conways recognized by the colors red and green. A 2-dimensional vector of Conway's functions is associated to each 2-tangle; the interior product is defined through the metric matrix M .

By using the colors it is interesting to note that the last families of the table are factorized in terms of 3-tangles because it is not possible to separate those families in 2-tangles formed, at least one of them, by two conways. The families with three colors do not allow to dissect in two 2-tangles of three conways as the other families of six conways; these families corresponding to one particular seed could be separated either with two colors (2-tangles formed by 2 and 4 conways) or with two colors (two 3-tangles formed by three conways).

Note that I have not developed very far the algebra of 3-tangles. The 5-dimensional vectors that appear in the factorizations of 3-tangles in this table may be identified as only four different vectors.

Many factorizations in this table could be modified in multiple forms by using the fact that the matrices of the form

$$\begin{pmatrix} A_1 & B_1 \\ B_1 & 0 \end{pmatrix}, \quad \begin{pmatrix} A_2 & B_2 \\ B_2 & 0 \end{pmatrix} \quad (3)$$

commute when multiplied with the metric M as it occurs in many terms of this table namely

$$\begin{pmatrix} A_1 & B_1 \\ B_1 & 0 \end{pmatrix} M \begin{pmatrix} A_2 & B_2 \\ B_2 & 0 \end{pmatrix} = \begin{pmatrix} A_2 & B_2 \\ B_2 & 0 \end{pmatrix} M \begin{pmatrix} A_1 & B_1 \\ B_1 & 0 \end{pmatrix}. \quad (4)$$

The transformation of this equation by the M matrix gives the equivalent expression which could be used to obtain other different factorizations

$$\begin{pmatrix} 0 & B_1 \\ B_1 & A_1 \end{pmatrix} M \begin{pmatrix} 0 & B_2 \\ B_2 & A_2 \end{pmatrix} = \begin{pmatrix} 0 & B_2 \\ B_2 & A_2 \end{pmatrix} M \begin{pmatrix} 0 & B_1 \\ B_1 & A_1 \end{pmatrix}. \quad (5)$$

Notice that these properties are related to the existence of abelian groups

$$\begin{pmatrix} A_1 & B_1 \\ B_1 & 0 \end{pmatrix} M \begin{pmatrix} A_2 & B_2 \\ B_2 & 0 \end{pmatrix} = \begin{pmatrix} A_1 B_2 + A_2 B_1 & B_1 B_2 \\ B_1 B_2 & 0 \end{pmatrix}, \quad (6)$$

and

$$\begin{pmatrix} 0 & B_1 \\ B_1 & A_1 \end{pmatrix} M \begin{pmatrix} 0 & B_2 \\ B_2 & A_2 \end{pmatrix} = \begin{pmatrix} 0 & B_1 B_2 \\ B_1 B_2 & A_1 B_2 + A_2 B_1 \end{pmatrix}. \quad (7)$$

Particular cases of this property at the beginning and ending of the factorization are remarked. We list a few examples at the beginning, but transposing these matrix equations you find similar expressions at the end of a product of factors.

$$\begin{aligned} \begin{pmatrix} A_1 & B_1 \end{pmatrix} M \begin{pmatrix} 0 & A_2 \\ A_2 & B_2 \end{pmatrix} &= \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & A_1 \\ A_1 & B_1 \end{pmatrix} M \begin{pmatrix} 0 & A_2 \\ A_2 & B_2 \end{pmatrix} = \\ &= \begin{pmatrix} A_2 & B_2 \end{pmatrix} M \begin{pmatrix} 0 & A_1 \\ A_1 & B_1 \end{pmatrix}. \end{aligned} \quad (8)$$

and

$$\begin{aligned} \begin{pmatrix} A_1 & B_1 \end{pmatrix} M \begin{pmatrix} A_2 & B_2 \\ B_2 & 0 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} A_1 & B_1 \\ B_1 & 0 \end{pmatrix} M \begin{pmatrix} A_2 & B_2 \\ B_2 & 0 \end{pmatrix} = \\ &= \begin{pmatrix} A_2 & B_2 \end{pmatrix} M \begin{pmatrix} A_1 & B_1 \\ B_1 & 0 \end{pmatrix}. \end{aligned} \quad (9)$$

References

- [1] J. H. Conway *An enumeration of knots and links and some of their algebraic properties*, in J. Leech (Ed.) *Computational Problems in Abstract Algebra*, (Pergamon Press, Oxford, 1970, pp. 329-358)
- [2] E. Piña (Preprint) arXiv:math.GN.3355/1206., 15 Jun 2012 *Algebra of families of alternating knots and tangles*
- [3] E. Piña (Preprint) arXiv:math.GT/0712.2229, 13 Dec 2007 *Conway classification of alternating knots*

2 THE FAMILIES OF KNOTS OF ONE AND TWO CONWAYS

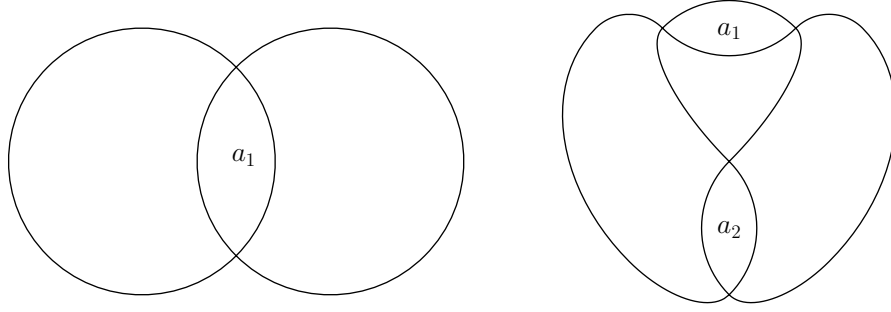


Figure 1:
Rational knots, its Conway's functions are a_1 and $1 + a_1a_2$.

THE FAMILIES OF KNOTS OF THREE CONWAYS WITH SEED THE TREFOIL TORUS KNOT 3_1 (2 CASES)

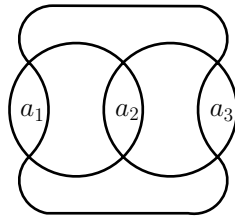


Figure 2:

$$a_1a_2 + a_2a_3 + a_3a_1 = (a_1, 1)M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} a_3 \\ 1 \end{pmatrix}$$

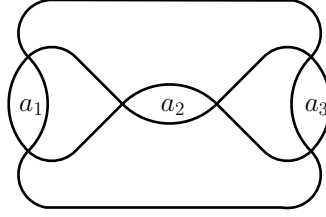


Figure 3:

$$a_1 a_2 a_3 + a_1 + a_3 = (a_1, 1) M \begin{pmatrix} 0 & 1 \\ 1 & a_2 \end{pmatrix} M \begin{pmatrix} a_3 \\ 1 \end{pmatrix}$$

3 THE FAMILIES OF KNOTS OF FOUR CONWAYS (5 CASES)

3.1 The families of knots with seed the Solomon link (two cases)

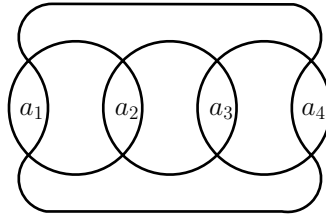


Figure 4:

$$(a_1 a_2, a_1 + a_2) M \begin{pmatrix} a_3 a_4 \\ a_3 + a_4 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} a_4 \\ 1 \end{pmatrix}$$

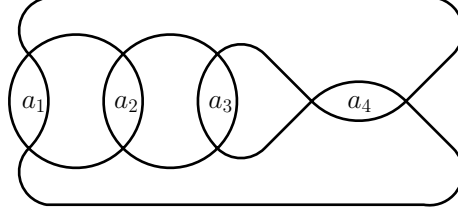


Figure 5:

$$(a_1 a_2, a_1 + a_2) M \begin{pmatrix} a_3 \\ a_3 a_4 + 1 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 1 \\ a_4 \end{pmatrix}$$

3.2 The families of knots with seed the knot 4_1 (three cases)

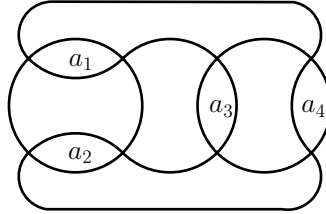


Figure 6:

$$(a_1 + a_2, a_1 a_2) M \begin{pmatrix} a_3 a_4 \\ a_3 + a_4 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} a_4 \\ 1 \end{pmatrix}$$

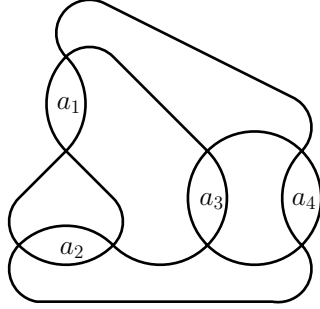


Figure 7:

$$(a_1 a_2 + 1, a_2) M \begin{pmatrix} a_3 a_4 \\ a_3 + a_4 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} a_4 \\ 1 \end{pmatrix}$$

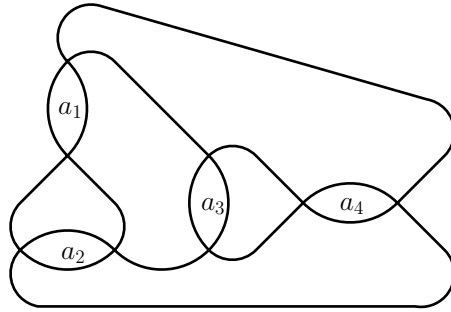


Figure 8:

$$(a_1 a_2 + 1, a_2) M \begin{pmatrix} a_3 \\ a_3 a_4 + 1 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 1 \\ a_4 \end{pmatrix}$$

4 THE FAMILIES OF KNOTS OF FIVE CONWAYS (12 CASES)

4.1 The families of knots with seed the torus knot 5_1 (two cases)

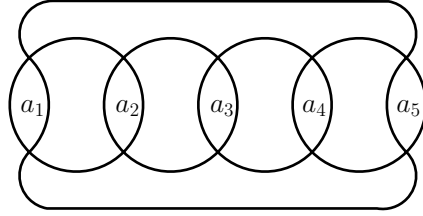


Figure 9:

$$(a_1 a_2 a_3, a_1 a_2 + a_2 a_3 + a_3 a_1) M \begin{pmatrix} a_4 a_5 \\ a_4 + a_5 \end{pmatrix} = (a_1 a_2, a_1 + a_2) M \begin{pmatrix} a_3 a_4 a_5 \\ a_3 a_4 + a_4 a_5 + a_5 a_3 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} a_5 \\ 1 \end{pmatrix}$$

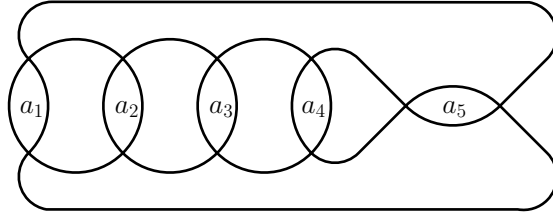


Figure 10:

$$(a_1 a_2 a_3, a_1 a_2 + a_2 a_3 + a_3 a_1) M \begin{pmatrix} a_4 \\ a_4 a_5 + 1 \end{pmatrix} = (a_1 a_2, a_1 + a_2) M \begin{pmatrix} a_3 a_4 \\ a_3 a_4 a_5 + a_3 + a_4 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}$$

4.2 The families of knots with seed the knot 5_2 (four cases)

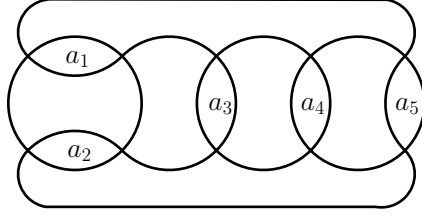


Figure 11:

$$(a_1+a_2, a_1a_2)M \begin{pmatrix} a_3a_4a_5 \\ a_3a_4 + a_4a_5 + a_5a_3 \end{pmatrix} = ((a_1+a_2)a_3, a_1a_2a_3+a_1+a_2)M \begin{pmatrix} a_4a_5 \\ a_4 + a_5 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} a_5 \\ 1 \end{pmatrix}$$

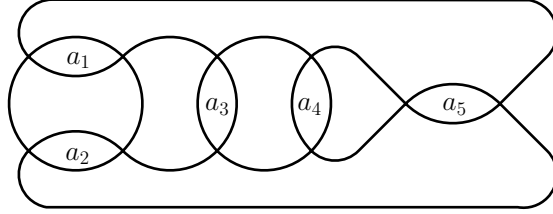


Figure 12:

$$((a_1+a_2)a_3, a_1a_2a_3+a_1+a_2)M \begin{pmatrix} a_4 \\ a_4a_5 + 1 \end{pmatrix} = (a_1+a_2, a_1a_2)M \begin{pmatrix} a_3a_4 \\ a_3a_4a_5 + a_3 + a_4 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}$$

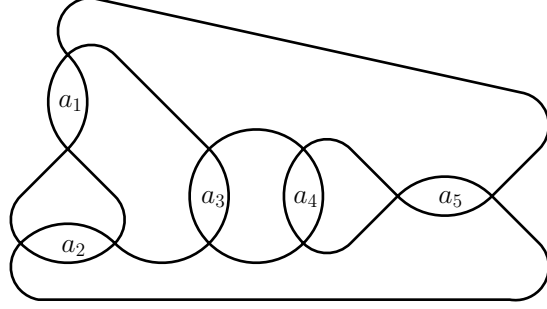


Figure 13:

$$(a_1 a_2 + 1, a_2) M \begin{pmatrix} a_3 a_4 \\ a_3 a_4 a_5 + a_3 + a_4 \end{pmatrix} = (a_1 a_2 a_3 + a_3, a_2(a_1 + a_3) + 1) M \begin{pmatrix} a_4 \\ a_4 a_5 + 1 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}$$

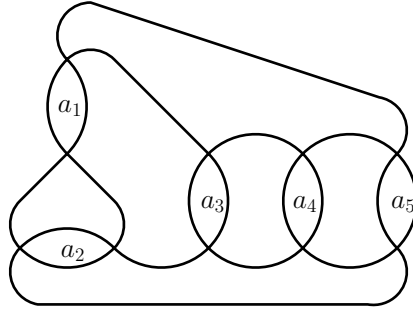


Figure 14:

$$(a_1 a_2 + 1, a_2) M \begin{pmatrix} a_3 a_4 a_5 \\ a_3 a_4 + a_4 a_5 + a_5 a_3 \end{pmatrix} = (a_1 a_2 a_3 + a_3, a_2(a_1 + a_3) + 1) M \begin{pmatrix} a_4 a_5 \\ a_4 + a_5 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} a_5 \\ 1 \end{pmatrix}$$

4.3 The families of knots with seed the Whitehead link 5_1^2 (six cases)

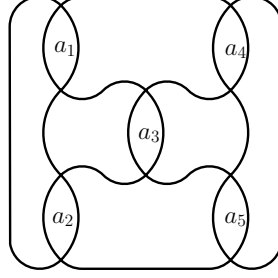


Figure 15:

$$(a_1 + a_2, a_1 a_2) M \begin{pmatrix} a_3(a_4 + a_5) \\ a_3 a_4 a_5 + a_4 + a_5 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_4 \\ a_4 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}$$

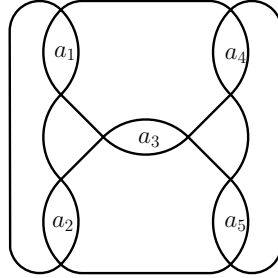


Figure 16:

$$(a_1 + a_2, a_1 a_2) M \begin{pmatrix} a_4 + a_5 \\ a_3 a_4 + a_4 a_5 + a_5 a_4 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_3 \end{pmatrix} M \begin{pmatrix} 1 & a_4 \\ a_4 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}$$

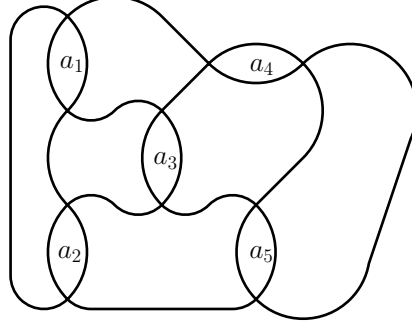


Figure 17:

$$\begin{aligned}
 (a_1+a_2, a_1a_2)M \begin{pmatrix} a_3(a_4a_5+1) \\ (a_3(a_4+a_5)+1) \end{pmatrix} &= ((a_1+a_2)a_3, a_1a_2a_3+a_1+a_2)M \begin{pmatrix} a_4a_5+1 \\ a_5 \end{pmatrix} = \\
 (1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} a_4 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}
 \end{aligned}$$

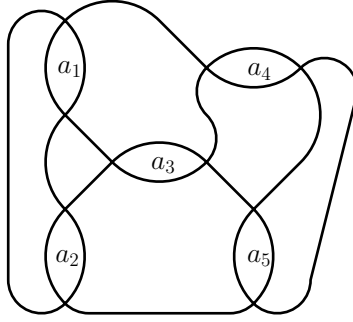


Figure 18:

$$\begin{aligned}
 (a_1+a_2, a_1a_2)M \begin{pmatrix} a_4a_5+1 \\ a_3a_4a_5+a_3+a_5 \end{pmatrix} &= (a_1+a_2, a_1a_2+a_2a_3+a_3a_1)M \begin{pmatrix} a_4a_5+1 \\ a_5 \end{pmatrix} = \\
 (1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_3 \end{pmatrix} M \begin{pmatrix} a_4 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}
 \end{aligned}$$

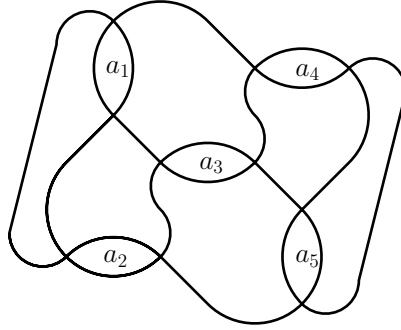


Figure 19:

$$(a_1 a_2 + 1, a_1) M \begin{pmatrix} a_4 a_5 + 1 \\ a_3 a_4 a_5 + a_3 + a_5 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} a_2 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_3 \end{pmatrix} M \begin{pmatrix} a_4 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}$$

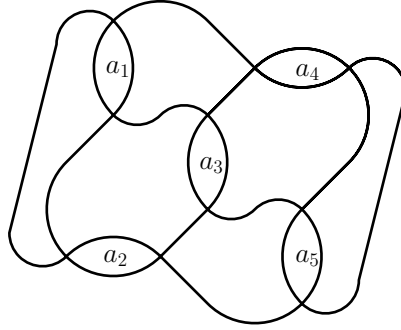


Figure 20:

$$(a_1 a_2 + 1, a_1) M \begin{pmatrix} a_3(a_4 a_5 + 1) \\ a_5(a_3 + a_4) + 1 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} a_2 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} a_4 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}$$

5 THE FAMILIES OF KNOTS OF SIX CONWAYS (44 CASES)

5.1 The families of knots with seed the torus link 6_2^1 (two cases)

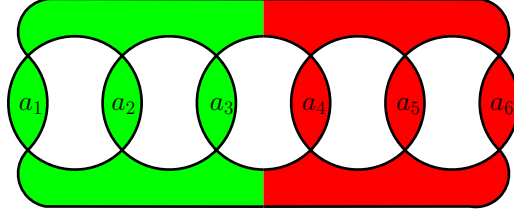


Figure 21:

$$(a_1 a_2 a_3, a_1 a_2 + a_2 a_3 + a_3 a_1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4 a_5 a_6 \\ a_4 a_5 + a_5 a_6 + a_6 a_4 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_5 \\ a_5 & 1 \end{pmatrix} M \begin{pmatrix} a_6 \\ 1 \end{pmatrix}$$

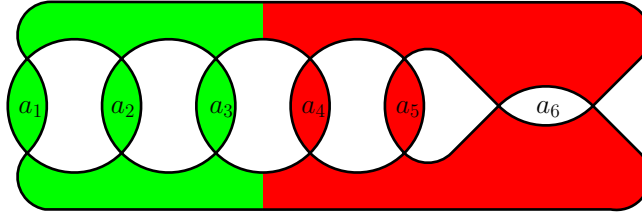


Figure 22:

$$(a_1 a_2 a_3, a_1 a_2 + a_2 a_3 + a_3 a_1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4 a_5 \\ a_4 a_5 a_6 + a_4 + a_5 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_5 \\ a_5 & 1 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

5.2 Families of knots with seed the knot 6_1 (four cases)

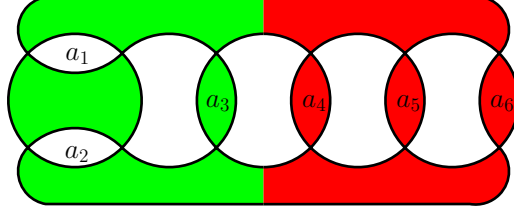


Figure 23:

$$((a_1 + a_2)a_3, a_1a_2a_3 + a_1 + a_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4a_5a_6 \\ a_4a_5 + a_5a_6 + a_6a_4 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_5 \\ a_5 & 1 \end{pmatrix} M \begin{pmatrix} a_6 \\ 1 \end{pmatrix}$$

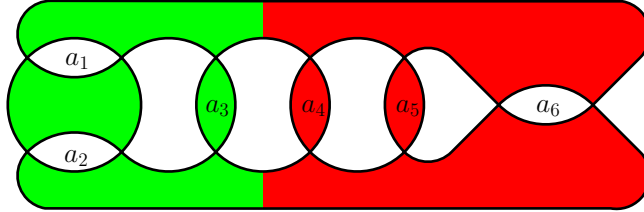


Figure 24:

$$(a_1 + a_2)a_3, (a_1a_2a_3 + a_1 + a_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4a_5 \\ a_4a_5a_6 + a_4 + a_5 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_5 \\ a_5 & 1 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

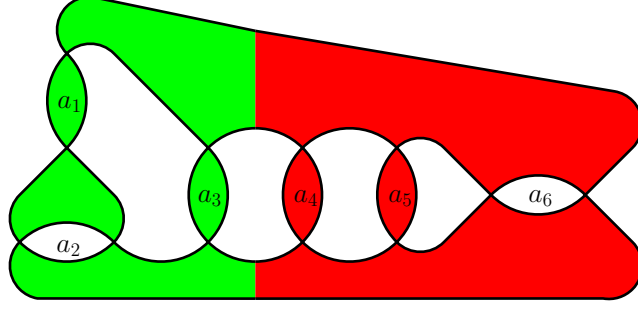


Figure 25:

$$((a_1a_2 + 1)a_3, a_2(a_1 + a_3) + 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4a_5 \\ a_4a_5a_6 + a_4 + a_5 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_5 \\ a_5 & 1 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

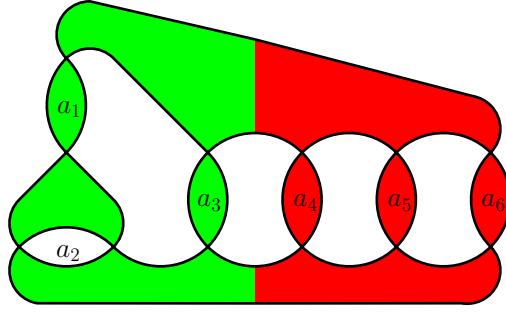


Figure 26:

$$((a_1a_2 + 1)a_3, a_2(a_1 + a_3) + 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4a_5a_6 \\ a_4a_5 + a_5a_6 + a_6a_4 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_5 \\ a_5 & 1 \end{pmatrix} M \begin{pmatrix} a_6 & \\ & 1 \end{pmatrix}$$

5.3 Families of knots with seed the link 6_2^2 (three cases)

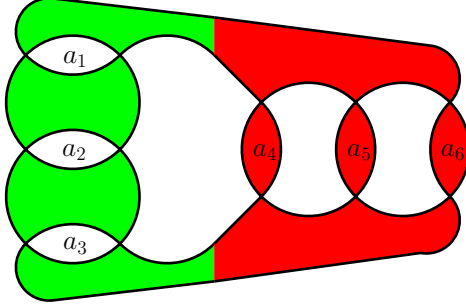


Figure 27:

$$(a_1a_2 + a_2a_3 + a_3a_1, a_1a_2a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4a_5a_6 \\ a_4a_5 + a_5a_6 + a_6a_4 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_5 \\ a_5 & 1 \end{pmatrix} M \begin{pmatrix} a_6 \\ 1 \end{pmatrix}$$

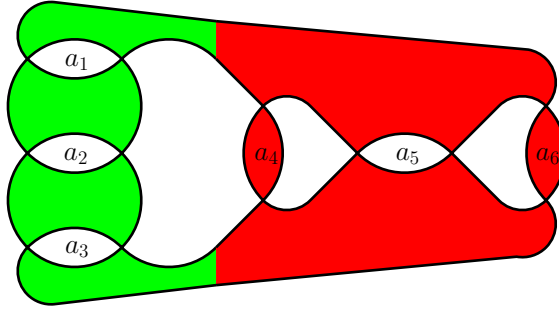


Figure 28:

$$(a_1a_2 + a_2a_3 + a_3a_1, a_1a_2a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4a_6 \\ a_4a_5a_6 + a_4 + a_6 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_5 \end{pmatrix} M \begin{pmatrix} a_6 \\ 1 \end{pmatrix}$$

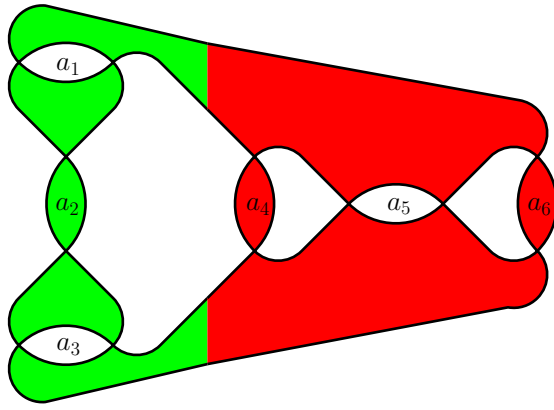


Figure 29:

$$\begin{aligned}
 & (a_1 a_2 a_3 + a_1 + a_3, a_1 a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4 a_6 \\ a_4 a_5 a_6 + a_4 + a_6 \end{pmatrix} = \\
 & (1, a_1) M \begin{pmatrix} a_2 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_5 \end{pmatrix} M \begin{pmatrix} a_6 \\ 1 \end{pmatrix}
 \end{aligned}$$

5.4 Families of knots with seed the link 6_3^2 with twelve terms (six cases)

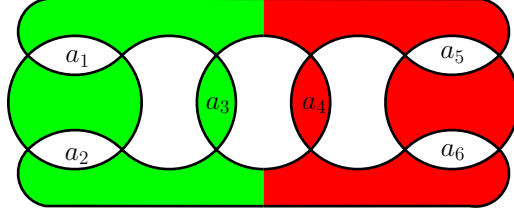


Figure 30:

$$((a_1 + a_2)a_3, a_1a_2a_3 + a_1 + a_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5 + a_6) \\ a_4a_5a_6 + a_5 + a_6 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

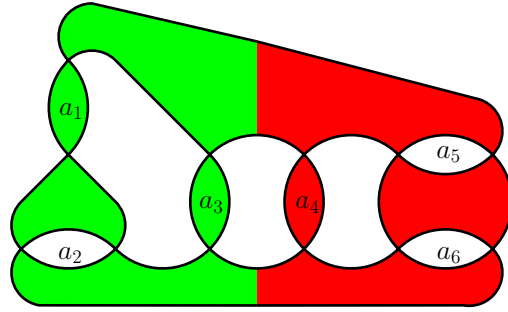


Figure 31:

$$((a_1a_2 + 1)a_3, a_2(a_1 + a_3) + 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5 + a_6) \\ a_4a_5a_6 + a_5 + a_6 \end{pmatrix} =$$

$$(a_1, 1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

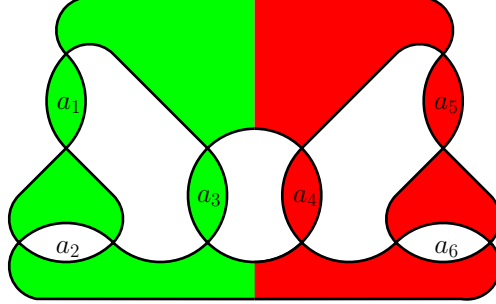


Figure 32:

$$((a_1a_2 + 1)a_3, a_2(a_1 + a_3) + 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5a_6 + 1) \\ (a_4 + a_5)a_6 + 1 \end{pmatrix} =$$

$$(a_1, 1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} a_5 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

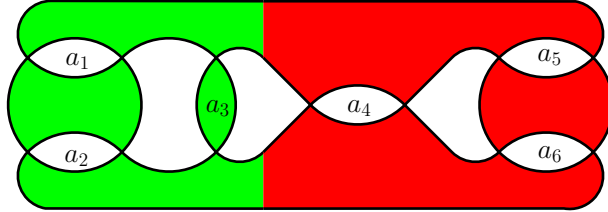


Figure 33:

$$((a_1 + a_2)a_3, a_1a_2a_3 + a_1 + a_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_4a_5 + a_5a_6 + a_6a_4 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

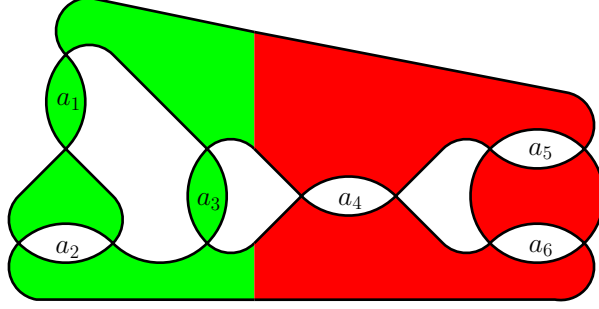


Figure 34:

$$((a_1a_2 + 1)a_3, a_2(a_1 + a_3) + 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_4a_5 + a_5a_6 + a_6a_4 \end{pmatrix} =$$

$$(a_1, 1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

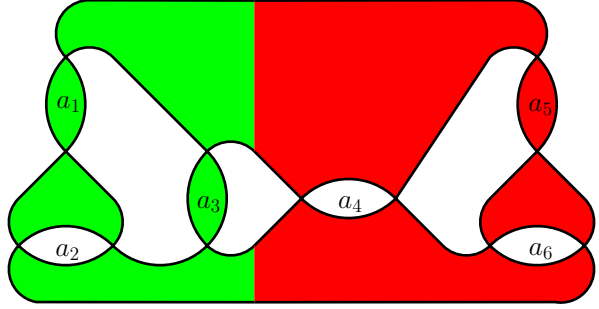


Figure 35:

$$((a_1a_2 + 1)a_3, a_2(a_1 + a_3) + 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5a_6 + 1 \\ a_4a_5a_6 + a_4 + a_6 \end{pmatrix} =$$

$$(a_1, 1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 \\ a_3 & 1 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} a_5 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

5.5 Families of knots with seed the knot 6_2 , the functions have eleven terms (eight cases)

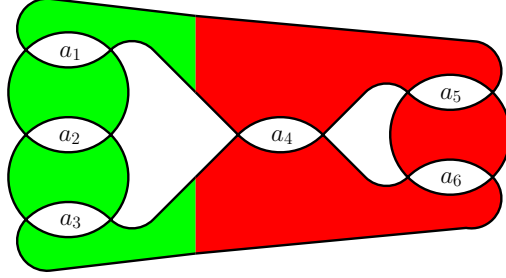


Figure 36:

$$(a_1a_2 + a_2a_3 + a_3a_1, a_1a_2a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_4a_5 + a_5a_6 + a_6a_4 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

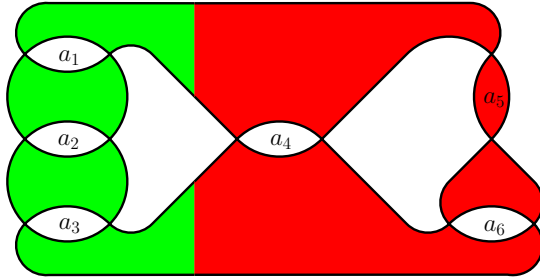


Figure 37:

$$(a_1a_2 + a_2a_3 + a_3a_1, a_1a_2a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5a_6 + 1 \\ a_4a_5a_6 + a_4 + a_6 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} a_5 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

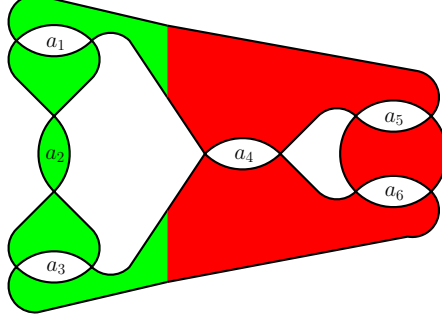


Figure 38:

$$(a_1 a_2 a_3 + a_1 + a_3, a_1 a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_4 a_5 + a_5 a_6 + a_6 a_4 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} a_2 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

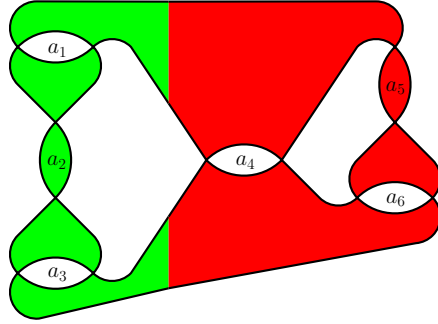


Figure 39:

$$(a_1 a_2 a_3 + a_1 + a_3, a_1 a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 a_6 + 1 \\ a_4 a_5 a_6 + a_4 + a_6 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} a_2 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} a_5 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

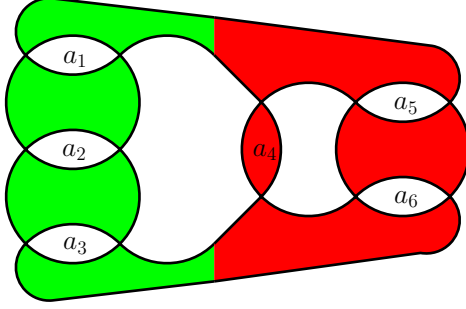


Figure 40:

$$(a_1a_2 + a_2a_3 + a_3a_1, a_1a_2a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5 + a_6) \\ a_4a_5a_6 + a_5 + a_6 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

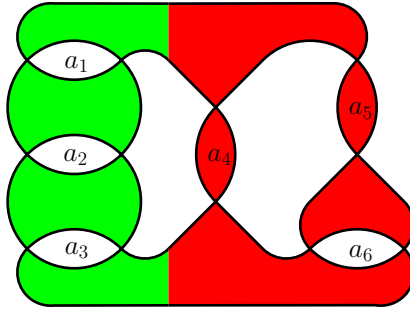


Figure 41:

$$(a_1a_2 + a_2a_3 + a_3a_1, a_1a_2a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5a_6 + 1) \\ (a_4 + a_5)a_6 + 1 \end{pmatrix} =$$

$$(1, a_1)M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} a_5 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

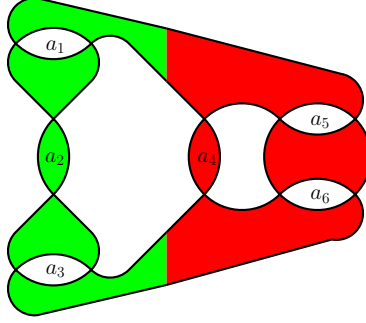


Figure 42:

$$(a_1 a_2 a_3 + a_1 + a_3, a_1 a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5 + a_6) \\ a_4 a_5 a_6 + a_5 + a_6 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} a_2 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

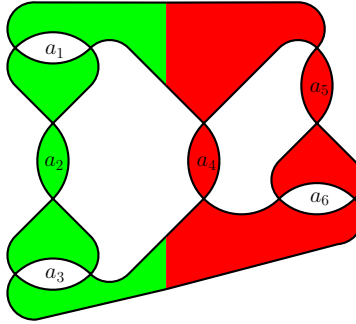


Figure 43:

$$(a_1 a_2 a_3 + a_1 + a_3, a_1 a_3) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5 a_6 + 1) \\ (a_4 + a_5)a_6 + 1 \end{pmatrix} =$$

$$(1, a_1) M \begin{pmatrix} a_2 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} a_5 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

5.6 Families of knots with seed the knot 6_3 , the function of Conway has thirteen terms (ten cases)

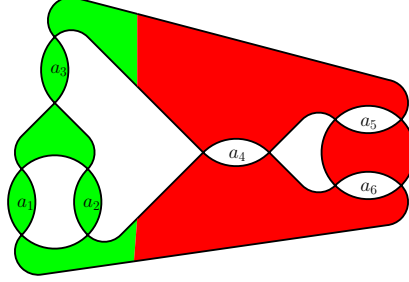


Figure 44:

$$(a_1a_2 + a_2a_3 + a_3a_1, a_1 + a_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_4a_5 + a_5a_6 + a_6a_4 \end{pmatrix} =$$

$$(a_1, 1)M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} a_3 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

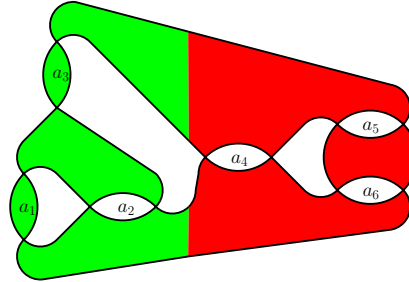


Figure 45:

$$(a_1a_2a_3 + a_1 + a_3, a_1a_2 + 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_4a_5 + a_5a_6 + a_6a_4 \end{pmatrix} =$$

$$(a_1, 1)M \begin{pmatrix} 0 & 1 \\ 1 & a_2 \end{pmatrix} M \begin{pmatrix} a_3 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

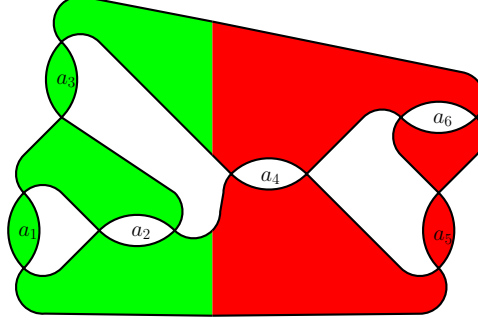


Figure 46:

$$(a_1 a_2 a_3 + a_1 + a_3, a_1 a_2 + 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 a_6 + 1 \\ a_4 a_5 a_6 + a_4 + a_6 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & 1 \\ 1 & a_2 \end{pmatrix} M \begin{pmatrix} a_3 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} a_5 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

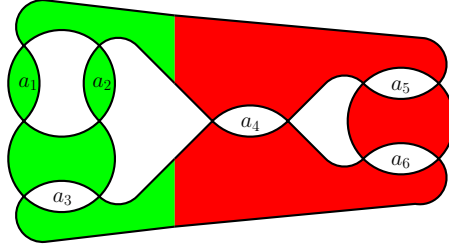


Figure 47:

$$(a_1 a_2 a_3 + a_1 + a_2, a_3(a_1 + a_2)) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_4 a_5 + a_5 a_6 + a_6 a_4 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

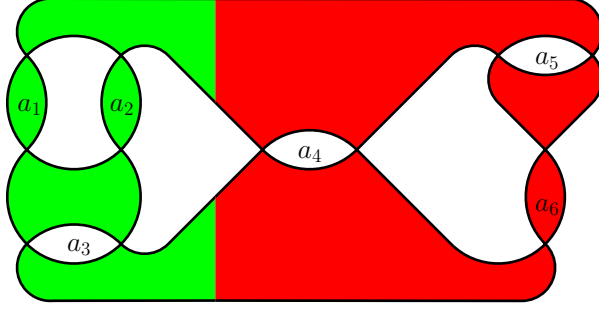


Figure 48:

$$(a_1 a_2 a_3 + a_1 + a_2, a_3(a_1 + a_2)) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 a_6 + 1 \\ a_4 a_5 a_6 + a_4 + a_5 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} a_6 \\ 1 \end{pmatrix}$$

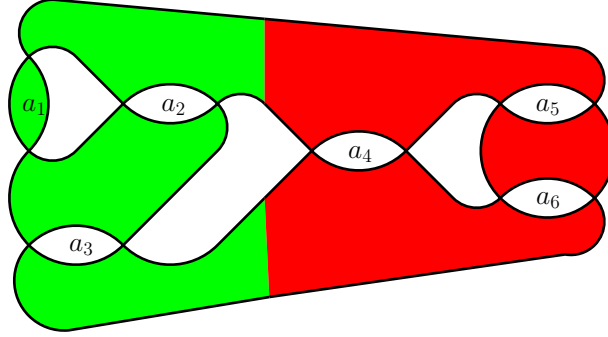


Figure 49:

$$(a_1(a_2 + a_3) + 1, a_3(a_1 a_2 + 1)) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_4 a_5 + a_5 a_6 + a_6 a_4 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & 1 \\ 1 & a_2 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix}$$

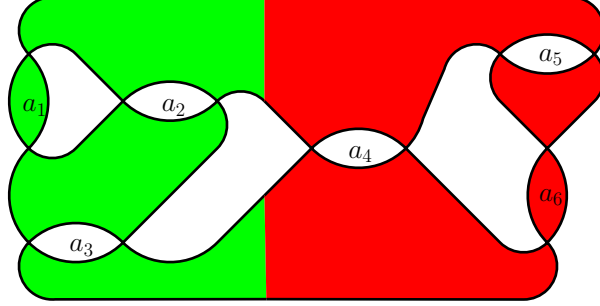


Figure 50:

$$\begin{aligned}
 & (a_1(a_2 + a_3) + 1, a_3(a_1 a_2 + 1)) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 a_6 + 1 \\ a_4 a_5 a_6 + a_4 + a_5 \end{pmatrix} = \\
 & (a_1, 1) M \begin{pmatrix} 0 & 1 \\ 1 & a_2 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & 1 \\ 1 & a_4 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} a_6 & \\ & 1 \end{pmatrix}
 \end{aligned}$$

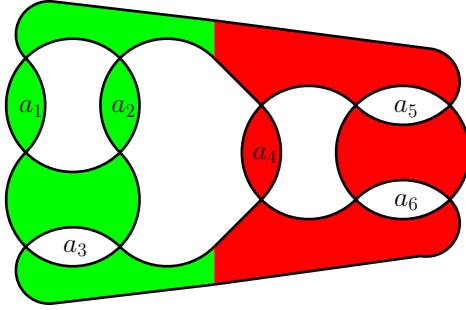


Figure 51:

$$\begin{aligned}
 & (a_1 a_2 a_3 + a_1 + a_2, a_3(a_1 + a_2)) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5 + a_6) \\ a_4 a_5 a_6 + a_5 + a_6 \end{pmatrix} = \\
 & (a_1, 1) M \begin{pmatrix} 0 & a_2 \\ a_2 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}
 \end{aligned}$$

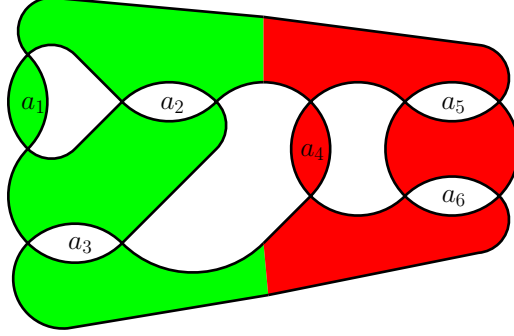


Figure 52:

$$(a_1(a_2 + a_3) + 1, a_3(a_1a_2 + 1)) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5 + a_6) \\ a_4a_5a_6 + a_5 + a_6 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & 1 \\ 1 & a_2 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

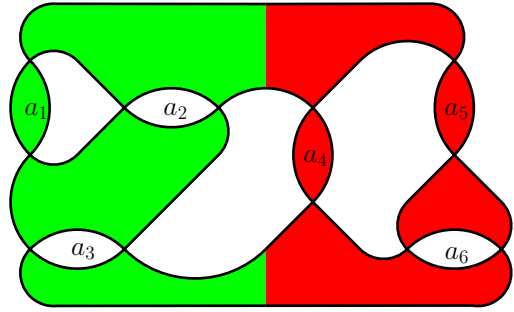


Figure 53:

$$(a_1(a_2 + a_3) + 1, a_3(a_1a_2 + 1)) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_4(a_5a_6 + 1) \\ (a_4 + a_5)a_6 + 1 \end{pmatrix} =$$

$$(a_1, 1) M \begin{pmatrix} 0 & 1 \\ 1 & a_2 \end{pmatrix} M \begin{pmatrix} 1 & a_3 \\ a_3 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_4 \\ a_4 & 1 \end{pmatrix} M \begin{pmatrix} a_5 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 & \\ & a_6 \end{pmatrix}$$

5.7 Families of knots with seed the link 6_1^3 (four cases)

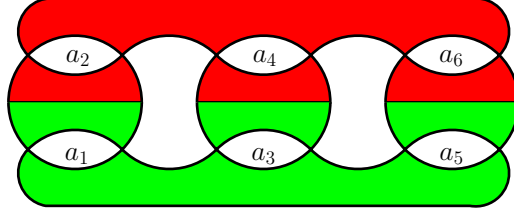


Figure 54:

$$\begin{pmatrix} a_1 a_3 a_5 \\ a_3 a_5 \\ a_5 a_1 \\ a_1 a_3 \\ a_1 + a_3 + a_5 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 a_4 a_6 \\ a_4 a_6 \\ a_6 a_2 \\ a_2 a_4 \\ a_2 + a_4 + a_6 \end{pmatrix}$$

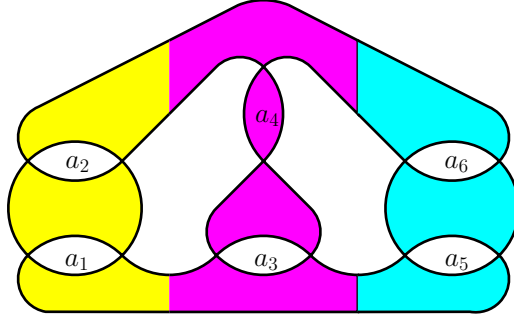


Figure 55:

$$\begin{aligned} & (a_1 + a_2, a_1 a_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & a_3 a_4 + 1 \\ a_3 a_4 + 1 & a_3 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 + a_6 \\ a_5 a_6 \end{pmatrix} = \\ & (1, a_1) M \begin{pmatrix} 1 & a_2 \\ a_2 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 a_4 + 1 \\ a_3 a_4 + 1 & a_3 \end{pmatrix} M \begin{pmatrix} 1 & a_5 \\ a_5 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_6 \end{pmatrix} \end{aligned}$$

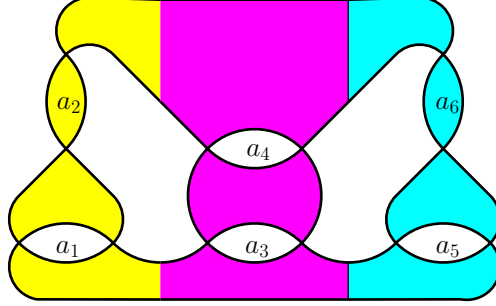


Figure 56:

$$\begin{aligned}
 & (a_1 a_2 + 1, a_1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & a_3 + a_4 \\ a_3 + a_4 & a_3 a_4 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_5 a_6 + 1 \\ a_5 \end{pmatrix} = \\
 & (1, a_1) M \begin{pmatrix} a_2 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 0 & a_3 + a_4 \\ a_3 + a_4 & a_3 a_4 \end{pmatrix} M \begin{pmatrix} a_6 & 1 \\ 1 & 0 \end{pmatrix} M \begin{pmatrix} 1 \\ a_5 \end{pmatrix}
 \end{aligned}$$

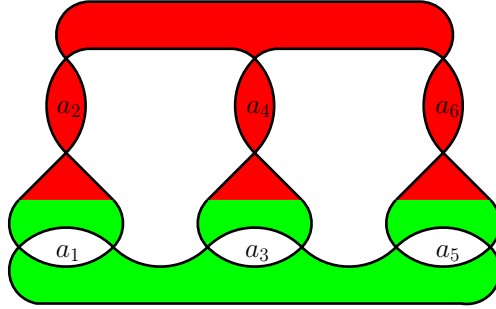


Figure 57:

$$\begin{pmatrix} a_1 a_3 a_5 \\ a_3 a_5 \\ a_5 a_1 \\ a_1 a_3 \\ a_1 + a_3 + a_5 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ a_2 \\ a_4 \\ a_6 \\ a_2 a_4 + a_4 a_6 + a_6 a_2 \end{pmatrix}$$

5.8 Families of knots with seed the Borromean link C_2^3
with sixteen terms (seven cases)

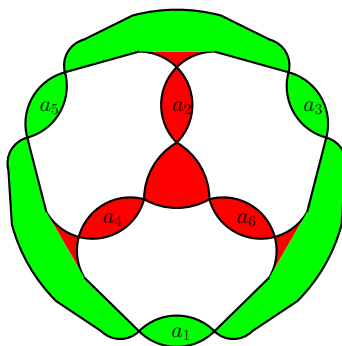


Figure 58:

$$\begin{pmatrix} a_1 + a_3 + a_5 \\ a_3 a_5 \\ a_5 a_1 \\ a_1 a_3 \\ a_1 a_3 a_5 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ a_2 \\ a_4 \\ a_6 \\ a_2 a_4 + a_4 a_6 + a_6 a_2 \end{pmatrix}$$

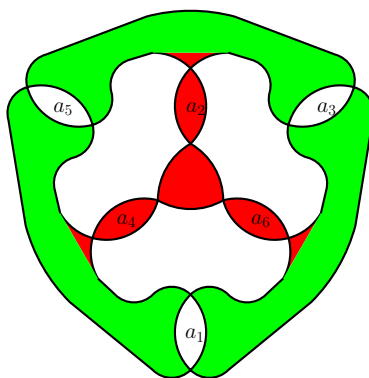


Figure 59:

$$\begin{pmatrix} a_1 a_3 + a_3 a_5 + a_5 a_1 \\ a_1 \\ a_3 \\ a_5 \\ 1 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ a_2 \\ a_4 \\ a_6 \\ a_2 a_4 + a_4 a_6 + a_6 a_2 \end{pmatrix}$$

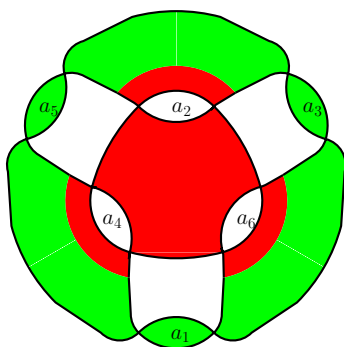


Figure 60:

$$\begin{pmatrix} a_1 + a_3 + a_5 \\ a_3 a_5 \\ a_5 a_1 \\ a_1 a_3 \\ a_1 a_3 a_5 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 a_4 a_6 \\ a_4 a_6 \\ a_6 a_2 \\ a_2 a_4 \\ a_2 + a_4 + a_6 \end{pmatrix}$$

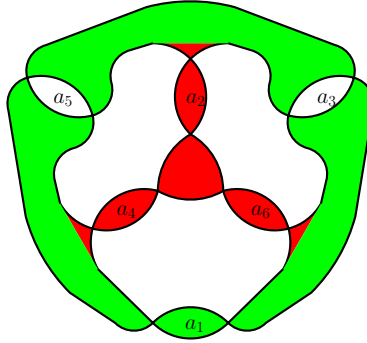


Figure 61:

$$\begin{pmatrix} a_1 a_3 a_5 + a_3 + a_5 \\ 1 \\ a_1 a_3 \\ a_1 a_5 \\ a_1 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ a_2 \\ a_4 \\ a_6 \\ a_2 a_4 + a_4 a_6 + a_6 a_2 \end{pmatrix} = \\
 \begin{pmatrix} 1 + a_2 a_3 + a_2 a_5 \\ a_2 a_3 a_5 \\ a_5 \\ a_3 \\ a_3 a_5 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_1 a_4 a_6 \\ a_4 a_6 \\ a_6 a_1 \\ a_1 a_4 \\ a_1 + a_4 + a_6 \end{pmatrix}$$

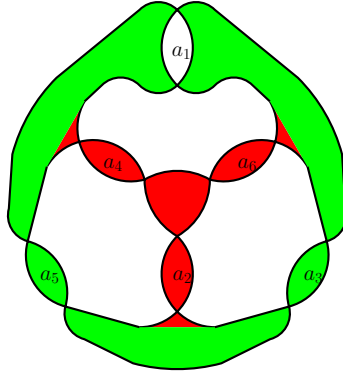


Figure 62:

$$\begin{pmatrix} a_1 a_3 + a_1 a_5 + 1 \\ a_1 a_3 a_5 \\ a_5 \\ a_3 \\ a_3 a_5 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ a_2 \\ a_4 \\ a_6 \\ a_2 a_4 + a_4 a_6 + a_6 a_2 \end{pmatrix} = \\
 \begin{pmatrix} a_1 a_3 a_6 + a_3 + a_6 \\ 1 \\ a_1 a_3 \\ a_1 a_6 \\ a_1 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_2 a_4 a_5 \\ a_4 a_5 \\ a_5 a_2 \\ a_2 a_4 \\ a_2 + a_4 + a_5 \end{pmatrix}$$

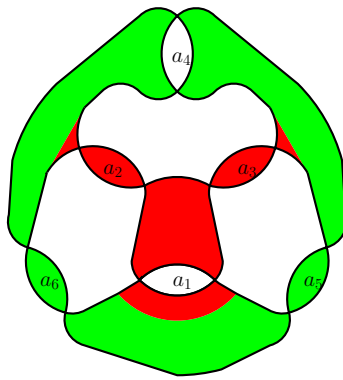


Figure 63:

$$\begin{pmatrix} a_4 a_5 + a_4 a_6 + 1 \\ a_5 \\ a_4 a_5 a_6 \\ a_6 \\ a_5 a_6 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_1 a_3 \\ 1 \\ a_1 a_2 \\ a_1 a_2 a_3 + a_2 + a_3 \end{pmatrix}$$

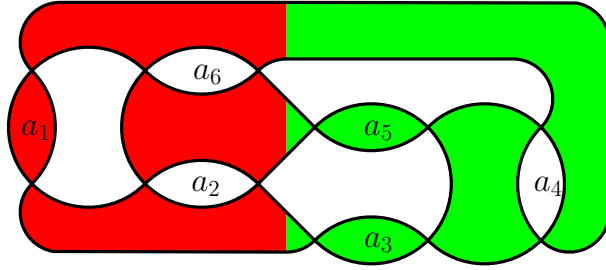


Figure 64:

$$\begin{pmatrix} a_1 a_2 a_6 + a_2 + a_6 \\ a_1 a_2 \\ a_1 a_6 \\ 1 \\ a_1 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_4 \\ 1 \\ a_3 a_4 \\ a_4 a_5 \\ a_3 a_4 a_5 + a_3 + a_5 \end{pmatrix} = \\
 \begin{pmatrix} a_2 a_5 + a_2 a_3 + 1 \\ a_2 a_3 a_5 \\ a_3 \\ a_5 \\ a_3 a_5 \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_4 a_6 \\ a_6 \\ a_1 a_4 a_6 \\ a_4 \\ a_1 a_4 + a_1 a_6 + 1 \end{pmatrix}$$